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EXPRESSING THE COEFFICIENTS OF THE EXPANSION OF A SCATTERING INDICATRIX THROUGH MIE COEFFICIENTS

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Equations are given expressing the coefficients of the expansion of a spherical-particle scattering indicatrix by Legendre polynomials directly through Mie coefficients. The equations are converted to a form suitable for use in a computer.

In some methods of solving the equation of radiant energy transfer the scattering indicatrix for a two-phase medium is represented in the form of a series by Legendre polynomials:

$$g(\mu) = 1 + \sum_{n=1}^{\infty} (2n + 1)\, g_n P_n(\mu). \tag{1}$$

The expansion coefficients g_n can be obtained from the known values $g(\mu)$ using the orthogonality properties of Legendre polynomials:

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$$g_n = \frac{1}{2}\int_{-1}^{+1} g(\mu) P_n(\mu)\, d\mu. \tag{2}$$

When determing f_n from Eq. (2), however, one must calculate the values of $g(\mu)$ for a large number of scattering angles. In the case of spherical particles the coefficients g_n can be calculated with the help of exact expressions obtained on the basis of the Mie equations [1]. At large values of the parameter ρ, however, the use of the equations presented in [1] is hindered owing to the necessity of calculating the factorials of large numbers. In addition, the coefficients A_n and B_n in [1] differ somewhat from the Mie coefficients a_n and b_n generally adopted in the current literature [2] and for which a convenient calculation algorithm has been developed [3]. For these reasons it is clearly preferable to use Eq. (2) [4].

Equations converted to a form more suitable for their use in a computer and expressing the coefficients of expansion g_n through a_n and b_n are given below.

We represent the coefficient of expansion $g_n(r)$ in the form

$$g_n(r) = \frac{\lambda^2}{2\pi}\frac{J_n(r)}{\sigma_{sca}(r)}, \tag{3}$$

where $\sigma_{sca}(r)$ is the scattering cross section, which is expressed through the coefficients a_n and b_n [2].

The value of $J_n(r)$ for a single particle is calculated from the following equation:

$$J_n(r) = \frac{(-1)^n}{2}\sum_{i=1}^{\infty}\frac{2i+1}{i(i+1)}\sum_{k=1}^{i}\frac{(-1)^{i+k}}{(1+\delta_{ik})}\frac{2k+1}{k(k+1)} W_{ik}\omega_{ikn}, \tag{4}$$

where $\omega_{ikn} = 0$ if $N = i + k - n < 0$ or $N > 2k + 1$. If $N = 2m$, $m = 0, 1, 2, \ldots, k$, then

$$W_{ik} = \mathrm{Re}(a_i)\,\mathrm{Re}(a_k) + \mathrm{Im}(a_i)\,\mathrm{Im}(a_k) + \mathrm{Re}(b_i)\,\mathrm{Re}(b_k) + \mathrm{Im}(b_i)\,\mathrm{Im}(b_k)$$

and

$$\omega_{ikn} = \frac{[i(i+1)+k(k+1)-n(n+1)]^2}{i+k+n+1}\prod_{\nu=1}^{\frac{i+n-k}{2}}\left(\frac{1+n-k}{2\nu}+1\right)\prod_{\nu=1}^{\frac{k+n-i}{2}}\left(\frac{k+n-i}{2\nu}+1\right)\prod_{\nu=1}^{n}\frac{1}{4}\left(\frac{N+3\nu+\frac{\nu^2}{N+\nu}}{N+n+\nu}\right).$$

If $N = 2m + 1$, $m = 0, 1, 2, \ldots, k$, then

$$W_{ik} = \mathrm{Re}(a_i)\,\mathrm{Re}(b_k) + \mathrm{Im}(a_i)\,\mathrm{Im}(b_k) + \mathrm{Re}(a_k)\,\mathrm{Re}(b_i) + \mathrm{Im}(a_k)\,\mathrm{Im}(b_i)$$

and

$$\omega_{ikn} = -N(i+n-k+1)(k+n-i+1)\prod_{\nu=1}^{\frac{i+n-k+1}{2}}\left(\frac{i+n-k+1}{2\nu}+1\right)\times\prod_{\nu=1}^{\frac{k+n-i+1}{2}}\left(\frac{k+n-i+1}{2\nu}+1\right)$$
$$\times\prod_{\nu=1}^{n+1}\frac{1}{4}\left(\frac{N-1+3\nu+\frac{\nu^2}{N+\nu}}{N+n+\nu}\right).$$

Let the particle-size distribution function $f(r)$ for a polydisperse system be given. Then the coefficients of the expansion of the scattering indicatrix for the polydisperse system are found through the values of $\sigma_{sca}(r)$ and $J_n(r)$ for a single particle:

$$g_n = \frac{\lambda^2}{2\pi}\frac{\int_0^{\infty} J_n(r) f(r)\, dr}{\int_0^{\infty}\sigma_{sca}(r) f(r)\, dr}. \tag{5}$$

Some values of the coefficients of the expansion of the scattering indicatrix for a single particle found from Eq. (3) are presented in Table 1.

TABLE 1. Values of Expansion Coefficients $g_n(\rho)$ of Scattering Indicatrices for Single Particles ($m = 1.8 - i \cdot 0.0001$)

ρ	n							
	1	2	3	4	5	6	7	8
1	0,2374	0,1154	0,0144	0,0010	0,0000	0,0000	0,0000	0,0000
2	0,5230	0,2414	0,1006	0,0358	0,0053	0,0006	0,0000	0,0000
3	0,5704	0,4723	0,2900	0,2080	0,1217	0,0617	0,0047	0,0008
4	0,4108	0,3346	0,1794	0,2085	0,1655	0,1476	0,0929	0,0592
5	0,3478	0,3644	0,9677	0,1649	0,0221	0,0980	0,0022	0,0881
10	0,6708	0,5894	0,4387	0,4613	0,3825	0,3973	0,3330	0,3259
20	0,7008	0,5904	0,4334	0,4483	0,3736	0,4059	0,3416	0,3691
30	0,7378	0,6500	0,5041	0,5256	0,4564	0,4954	0,4377	0,4745
40	0,7223	0,6240	0,4691	0,4939	0,4231	0,4639	0,4059	0 4462
50	0,7327	0,6550	0,5089	0,5438	0,4724	0,5177	0,4595	0,5050

For the distribution function $f(r) = 1.119 \cdot 3^{1.6} r^{0.6} \exp(-3r)$ [μm^{-1}] chosen as an example, with $\lambda = 2\ \mu$ and $m = 1.8 - i \cdot 0.0001$, the first three expansion coefficients calculated from Eq. (5) are $g_1 = 0.5228$, $g_2 = 0.3887$, and $g_3 = 0.2116$. The integration was done by Simpson's rule with a step $\Delta r = 0.05\ \mu m$. The corresponding coefficients determined from (2) with $\Delta r = 0.05\ \mu m$ and $\Delta\mu = 0.002$ are $g_1 = 0.5212$, $g_2 = 0.3870$, and $g_3 = 0.2109$.

In the given example 15 min was required to calculate the expansion coefficients g_n from Eq. (2) on an M-222 computer and only 1.6 min using Eq. (5).

NOTATION

$g(\mu)$	is the scattering indicatrix;
g_n	is the coefficient of the expansion of the scattering indicatrix in a series by Lengendre polynomials;
$P_n(\mu)$	is the Legendre polynomials of order n;
μ	is the cosine of scattering angle;
$\rho = 2\pi r/\lambda$	is the diffraction parameter;
λ	is the wavelength of radiation;
r	is the particle radius;
δ_{ik}	is the Kronecker symbol;
m	is the complex index of refraction of particle material;
Re(z) and Im(z)	are the real and imaginary parts of complex number z, respectively;
a_n and b_n	are the Mie coefficients;
$\sigma_{sca}(r)$	is the scattering cross section.

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